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STRUCTURAL DIVISION

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THE CONSTANT SEGMENT METHOD FOR THE ANALYSIS OF NON-UNIFORM STRUCTURAL MEMBERS

Walter E. Hanson,[1] A.M. ASCE, and Wallace F. Wiley[2]

SYNOPSIS

There are several general methods of analysis for beams having variable moments of inertia. Although precise, these methods are time consuming in application and subject to numerical errors. Even the combination of column analogy and moment distribution sometimes becomes quite laborious.

A method is presented in this paper which requires comparatively little computation, and which is simple in concept and use. It is based on the division of the flexural member into a number of segments of equal length, each segment being assumed to have a constant value of EI. In this respect the method may be considered an approximation; however, the accuracy afforded is very high, and even with slide-rule calculations it is entirely sufficient for all design purposes.

The discussion, examples, and tables presented herein cover only the computation of stiffness, carry-over factors, fixed end moments, and dead load deflections; but the method is applicable to other similar problems.

Basic Concepts

The behavior of any flexural member acted upon by any combination of transverse loads and end moments is dependent on the angle changes produced along the beam. These angle changes can be computed from the properties of the M/EI diagram.

For any non-uniform beam the shape of the M/EI diagram is irregular and may present difficulties in the computation of slopes and deflections. Such difficulties can be overcome by using the principle of superposition. The total slope or deflection at any one particular point is the summation of the slopes or deflections at that point produced by all segments of the M/EI diagram, each acting individually. While one segment is being considered, all other segments of the beam are assumed at that time to have an infinite value of EI. Figure 1 shows the behavior of a beam in which only the segment ef is assumed to be active.

If each segment may be considered to have a constant value of EI throughout its length, numerical computations can be greatly simplified. This is accomplished by the use of precalculated coefficients. These coefficients are the effects (end slopes, deflections at a particular point, etc.) of a segment of the moment diagram if this segment of the beam has a unit value of EI. In use, the coefficient for each segment is divided by the actual value of EI of that segment, and the results are summed across the beam to obtain the total effect.

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For purposes of computation, it is convenient to divide all beams into the same number of equal segments. The ten-segment scheme has been chosen for use in this paper. Each of the segments in this scheme is considered to have a constant value of EI throughout its length. If the value of EI changes within the segment, the weighted average value of 1/EI in the segment is used. That is, if the section changes abruptly at the 0.3 point of the segment, the weighted average value of 1/I may be taken as 0.3 of the value to the left plus 0.7 of the value to the right of the point of change.

Nomenclature and Conventions

The following symbols are used in the equations given in this paper:

a	ratio of distance to span length measured from the left support A
b	ratio of distance to span length measured from the left support to point of application of load P
c	ratio of distance to span length measured from left support to point of deflection
C_1--C_{10}	segmental coefficients for end slopes or deflections given in Tables I to VI of the Appendix
C_{AB}	moment carry-over factor from end A to end B
E	modulus of elasticity of beam material
I	moment of inertia of beam section
I_o	reference moment of inertia of beam sections
K_{AB}	stiffness factor at end A of span AB if end B is simply supported
K'_{AB}	stiffness factor at end A of span AB if end B is fixed
L	length of span
M_A^F	fixed end moment at end A of span AB
P	concentrated load
w	uniformly distributed load
Δ_D	deflection of beam at point D
θ_A^A	slope at end A of beam caused by a unit moment applied at A
θ_B^A	slope at end B of beam caused by a unit moment applied at A
θ_B^B	slope at end B of beam caused by a unit moment applied at B
θ_A^P	slope at end A of beam caused by a unit concentrated load
θ_A^w	slope at end A of beam caused by a unit uniformly distributed load

No sign convention is followed in this paper. The solution of any one of the equations will result in a positive quantity. The application of the proper sign is left to the user.

End Slopes Produced by Unit End Moments

The slope at the end of a beam is numerically equal to the reaction of the M/EI diagram at that end. If a unit moment is applied to end A of the beam AB, which has a constant value of EI, the M/EI diagram is as shown in Figure 2. Then, if only the shaded segment s is assumed to have a finite value of EI, the end slopes due to this segment are the reaction of the segment s of the M/EI diagram.

$$\theta_A^A = \frac{L}{EI}\left[a - a^2 + \frac{a^3}{3}\right] \qquad (1)$$

$$\theta_B^A = \frac{L}{EI}\left[\frac{a^2}{2} - \frac{a^3}{3}\right] \qquad (2)$$

Similarly, for a unit moment applied at B

$$\theta_B^B = \frac{L}{EI}\left[\frac{a^3}{3}\right] \qquad (3)$$

$$\theta_A^B = \frac{L}{EI}\left[\frac{a^2}{2} - \frac{a^3}{3}\right] \qquad (4)$$

It should be noted that equations (2) and (4) are identical. By Maxwell's law of reciprocal relations, this is true for any beam, no matter how the value of EI varies. That is,

$$\theta_B^A = \theta_A^B \qquad (5)$$

The quantities in brackets in these equations are coefficients of end slopes produced by a unit moment applied to the end of a beam having a constant value of EI. They represent, however, the slopes caused only by that segment of the M/EI diagram extending a distance aL along the beam. That is, all of the beam except the segment aL is considered to have an infinite value of EI.

The value of a coefficient for a segment extending from a_1L to a_2L is the difference between the values obtained by substituting successively for a the values of a_1 and a_2 in the bracketed factor. Values of these coefficients for ten segments of equal length were so obtained and are given in Table I of the Appendix.

When the value of EI is not constant throughout the length of the beam, the true value of an end slope is obtained by summation, thus

$$\theta_A^A = L \left[\frac{C_1}{EI_1} + \frac{C_2}{EI_2} + \cdots\cdots + \frac{C_{10}}{EI_{10}} \right]$$

Since E is usually a constant it can be placed outside the brackets. Also, for convenience in computation, if some value I_0 of moment of inertia is taken as a reference value, then

$$\theta_A^A = \frac{L}{EI_0} \left[C_1 \frac{I_0}{I_1} + C_2 \frac{I_0}{I_2} + \cdots\cdots + C_{10} \frac{I_0}{I_{10}} \right] \tag{6}$$

Equation (6) represents the general procedure used in this paper for evaluating the properties and behaviors of beams that have non-uniform sections.

Carry-over and Stiffness Factors

The moment carry-over factor from end A to end B of a beam is defined as the moment produced at the fixed end B by a unit moment applied at A. It can be shown that

$$C_{AB} = \theta_B^A / \theta_B^B \tag{7}$$

and

$$C_{BA} = \theta_A^B / \theta_A^A \tag{8}$$

Also, the stiffness factor at the end of a beam is defined as the moment required to rotate that end through a unit angle. From this definition, when end B is simply supported

$$K_{AB} = 1 / \theta_A^A \tag{9}$$

When end B is fixed

$$K'_{AB} = \frac{\theta_B^B}{\theta_A^A \theta_B^B - \theta_B^A \theta_A^B} \tag{10}$$

When end A is simply supported

$$K_{BA} = 1 / \theta_B^B \tag{11}$$

When A is fixed

$$K'_{BA} = \frac{\theta_A^A}{\theta_A^A \theta_B^B - \theta_B^A \theta_A^B} \tag{12}$$

Thus, it is evident that after the values of the end slopes have been determined as indicated in the preceding Section, the carry-over and stiffness factors may be found by equations (7) to (12). Also, it should be noted that the units of equations (9) to (12) are such that the stiffness factors vary directly as the moment of inertia times the modulus of elasticity and inversely as the length of the beam.

Fixed End Moments

Figure 3 shows the M/EI diagram for a simple beam having a constant value of EI and acted upon by a unit concentrated load. For any segment extending a distance aL along the beam from the left support A, when $a < b$

$$\theta_A^P = \frac{L^2}{EI}\left[\left(\frac{a^2}{2} - \frac{a^3}{3}\right)(1-b)\right] \tag{13}$$

and

$$\theta_B^P = \frac{L^2}{EI}\left[\frac{a^3}{3}(1-b)\right] \tag{14}$$

When $a > b$

$$\theta_A^P = \frac{L^2}{EI}\left[b(a - a^2 + \frac{a^3}{3}) - \frac{b^2}{2} + \frac{b^3}{6}\right] \qquad (15)$$

and

$$\theta_B^P = \frac{L^2}{EI}\left[b(\frac{a^2}{2} - \frac{a^3}{3}) - \frac{b^3}{6}\right] \qquad (16)$$

The quantities in brackets are coefficients for end slopes produced by a segment of the M/EI diagram extending a distance aL from end A. Coefficients for ten equal segments have been evaluated from these equations and are given in Table II of the Appendix. By the use of these coefficients the total values of end slopes for beams with variable moments of inertia may be found by a summation process similar to that indicated by equation (6).

For a unit uniform load, Figure 4

$$\theta_A^W = \frac{L^3}{EI}\left[\frac{a^2}{4} - \frac{a^3}{3} + \frac{a^4}{8}\right] \qquad (17)$$

and

$$\theta_B^W = \frac{L^3}{EI}\left[\frac{a^3}{6} - \frac{a^4}{8}\right] \qquad (18)$$

The coefficients of these slopes for ten equal segments are given in Table III of the Appendix.

With the end slopes evaluated, the following equations may be used to determine the fixed end moments. For concentrated loads, when both ends of the beam are fixed

$$M_A^F = P\left[\theta_A^P K'_{AB} - \theta_B^P C_{BA} K'_{BA}\right] \qquad (19)$$

and

$$M_B^F = P\left[\theta_B^P K'_{BA} - \theta_A^P C_{AB} K'_{AB}\right] \qquad (20)$$

For uniform loads, when both ends of the beam are fixed

$$M_A^F = W\left[\theta_A^W K'_{AB} - \theta_B^W C_{BA} K'_{BA}\right] \qquad (21)$$

and

$$M_B^F = W\left[\theta_B^W K'_{BA} - \theta_A^W C_{AB} K'_{AB}\right] \qquad (22)$$

For concentrated loads, when end A is simply supported

$$M_B^F = P\left[\theta_B^P K_{BA}\right] \qquad (23)$$

For uniform loads, when end A is simply supported

$$M_B^F = W\left[\theta_B^W K_{BA}\right] \qquad (24)$$

It should be noted that the units of the terms in the brackets of equations (19), (20), and (23) are such that the moments become the usual load times length. Similarly, moment equations (21), (22), and (24) reduce to the common form of load per unit length times length squared.

Dead Load Deflections

The dead load deflection at any point on a beam can also be computed by the method of constant EI segments. In this case, however, the quantities used are the moments of the segments of the M/EI diagram instead of the reactions.

In order to compute the dead load deflections of both simple and continuous beams, three types of moment diagrams may be considered: parabolic, triangular and rectangular. The parabolic diagram is used to compute the simple beam deflections, while the triangular and rectangular diagrams serve for computing the deflections due to end moments.

Figure 4 shows a beam having a constant value of EI subjected to a parabolic M/EI diagram resulting from a uniformly distributed load w per unit length. The deflection at point D caused by that segment of the M/EI diagram extending a distance aL along the beam from the left support A can be expressed by equation (25) when $a < d$

$$\Delta_D = \frac{WL^4}{EI}\left[\frac{1}{2}(1-d)\left(\frac{a^3}{3} - \frac{a^4}{4}\right)\right] \qquad (25)$$

When a > d

$$\Delta_D = \frac{WL^4}{EI}\left[\frac{d}{2}\left(\frac{a^2}{2} - \frac{2a^3}{3} + \frac{a^4}{4}\right) - \frac{d}{2}\left(\frac{d^2}{6} - \frac{d^3}{12}\right)\right] \qquad (26)$$

Similarly, for a moment M_T applied at end B, when a < d

$$\Delta_D = \frac{M_T L^2}{EI}\left[(1-d)\frac{a^3}{3}\right] \qquad (27)$$

When a > d

$$\Delta_D = \frac{M_T L^2}{EI}\left[d\left(\frac{a^2}{2} - \frac{a^3}{3}\right) - \frac{d^3}{6}\right] \qquad (28)$$

For a rectangular moment diagram with equal moments applied at ends A and B, when a < d

$$\Delta_D = \frac{M_R L^2}{EI}\left[(1-d)\frac{a^2}{2}\right] \qquad (29)$$

When a > d

$$\Delta_D = \frac{M_R L^2}{EI}\left[ad\left(1-\frac{a}{2}\right) - \frac{d^2}{2}\right] \qquad (30)$$

In each of these equations the quantities in brackets are coefficients of deflection of point D on a beam having a constant value of EI. These quantities are due only to the segment aL of the M/EI diagram. Coefficients are evaluated for ten equal segments in Tables IV, V, and VI of the Appendix for values of d of 1/4, 1/2 and 3/4.

For beams of non-uniform section the deflections are computed by the same process of summation as that indicated by equation (6) for end slopes.

Examples of Application

Figure 5 shows a three-span continuous beam and the loading for which the negative moments at supports B and C will be determined. Also, the deflection at the 0.5 point of span AB due to the uniform dead load will be found.

The variation in the moment of inertia and values of I_0/I for the end spans and center span are given in Figures 6 and 7. The average values of I_0/I for each of the 10 equal segments in each span are shown also in these figures.

Carry-over and Stiffness Factors

Figure 8 shows a typical form of the computations for determining the end slopes resulting from unit moments at end B of span AB and at both ends B

and C of span BC. The values of I_0/I are obtained from Figures 6 and 7 and the values of the coefficients are taken from Table I of the Appendix. The value of the coefficient for each segment is multiplied by the value of I_0/I for that segment and the product entered in the adjoining column. The columns are then totaled. The values of these column totals when multiplied by the ratio L/EI_0 give the end slopes.

Span AB is simply supported at A; therefore, the stiffness is found by equation (11). The beam is continuous over supports B and C; therefore, carry-over and stiffness factors for span BC are computed from equations (7), (8), (10), and (12). It should be noted that when the coefficients in Table I are used for the analysis of span BC, end A becomes B and end B becomes C. Because of symmetry the computations for span BC are reduced considerably, and the stiffness determined for span AB also applies to span CD.

Fixed End Moments

The fixed end moment at support B due to the concentrated and uniform load in span AB is computed in Figure 9. The coefficients for the end slope at B are obtained from Tables II and III of the Appendix. After the end slopes at B resulting from the concentrated and uniform loads have been determined, the respective fixed end moments are computed from equations (23) and (24).

In a similar manner in Figure 10, the fixed end moments are obtained at supports B and C for the loads in span BC. Since the span is fixed at both ends, the moments are found from equations (19), (20), (21), and (22). It should be noted that the computations are reduced because of symmetry of the uniform load and of the properties of the beam.

No computations for the fixed end moment in span CD are given because the moment may be determined by means of a ratio after the computations for the uniform load in span AB have been made.

Moment Distribution

The actual moments at supports B and C due to the total loading on the three-span continuous unit are computed in Figure 11(a). Similarly, the dead load moments are calculated in Figure 11(b). These computations should be self-explanatory to anyone familiar with the procedure of moment distribution.

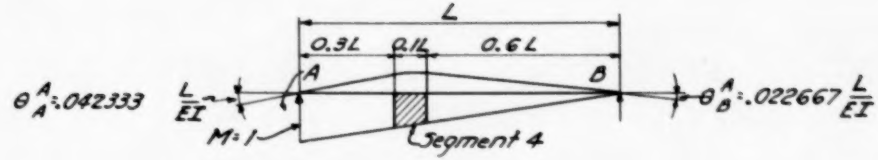
It is important to note that the computations for relative stiffnesses and distribution factors take into account the differences in span lengths. Since the same reference value of I_0 was used in determining the values of relative I in all spans and since E is constant, these values need not be included in the computations for the relative stiffnesses.

Deflection

The dead load deflection at the 0.5 point of span AB is found in Figure 12 using the coefficients from Tables IV and V of the Appendix. It should be noted that this deflection is made up of two parts. The deflection resulting from the simple beam, parabolic moment diagram is downward; while the deflection due to the triangular variation in negative moment, Figure 11(b), is upward.

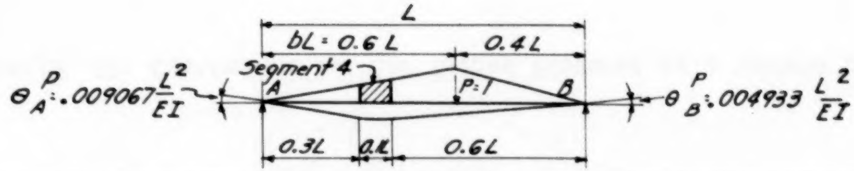
APPENDIX

Tables I to VI of this Appendix contain the segmental coefficients of end slopes and deflections of a non-uniform flexural member AB divided into ten equal segments. The sketch at the top of each table gives the key to the application and use of the coefficients given in the table.



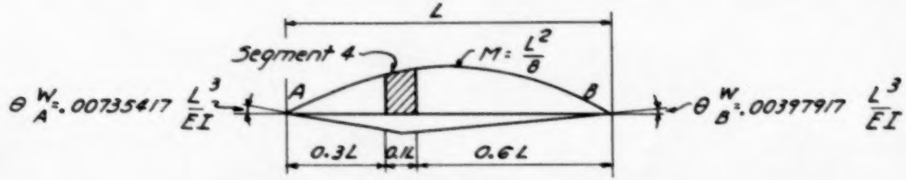
Segment	For θ_A^A	For θ_B^B	For $\theta_B^A = \theta_A^B$
1	.090333	.000333	.004667
2	.072333	.002333	.012667
3	.056333	.006333	.018667
4	.042333	.012333	.022667
5	.030333	.020333	.024667
6	.020333	.030333	.024667
7	.012333	.042333	.022667
8	.006333	.056333	.018667
9	.002333	.072333	.012667
10	.000333	.090333	.004667

TABLE I. Coefficients for End Slopes Produced by End Moments.



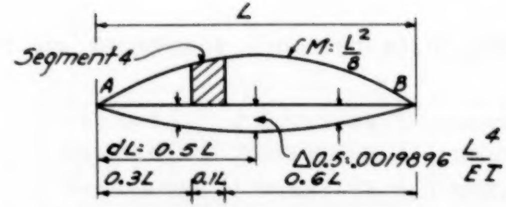
For θ_A^P										
Segment	Values of b									
	0.1	0.2	0.3	.0.4	.0.5	0.6	0.7	.0.8	.0.9	
1	.004200	.003733	.003267	.002800	.002333	.001867	.001400	.000933	.000467	10
2	.007233	.010133	.008867	.007600	.006333	.005067	.003800	.002533	.001267	9
3	.005633	.011267	.013067	.011200	.009333	.007467	.005600	.003733	.001867	8
4	.004233	.008467	.012700	.013600	.011333	.009067	.006800	.004533	.002267	7
5	.003033	.006067	.009100	.012133	.012333	.009867	.007400	.004933	.002467	6
6	.002033	.004067	.006100	.008133	.010167	.009867	.007400	.004933	.002467	5
7	.001233	.002467	.003700	.004933	.006167	.007400	.006800	.004533	.002267	4
8	.000633	.001267	.001900	.002533	.003167	.003800	.004433	.003733	.001867	3
9	.000233	.000467	.000700	.000933	.001167	.001400	.001633	.001867	.001267	2
10	.000033	.000067	000100	.000133	.000167	.000200	.000233	.000267	.000300	1
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	Segment
	Values of b									
	For θ_B^P									

TABLE II. Coefficients for End Slopes Produced by a Concentrated Load.



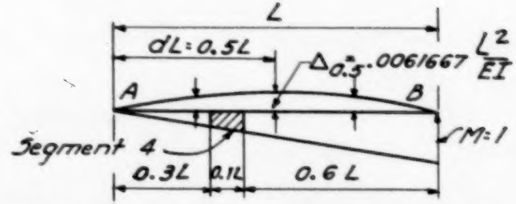
Segment	For θ_A^W	For θ_B^W
1	.00217917	.00015417
2	.00535417	.00097917
3	.00697917	.00235417
4	.00735417	.00397917
5	.00677917	.00555417
6	.00555417	.00677917
7	.00397917	.00735417
8	.00235417	.00697917
9	.00097917	.00535417
10	.00015417	.00217917

TABLE III. Coefficients for End Slopes Produced by a Uniform Load.



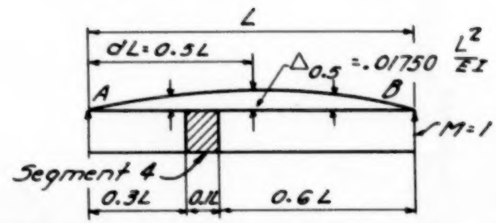
Segment	For d: 0.25	For d: 0.50	For d: 0.75
1	.0001156250	.0000770833	.0000385417
2	.0007343750	.0004895833	.0002447917
3	.0016388021	.0011770833	.0005885417
4	.0018385417	.0019895833	.0009947917
5	.0016947917	.0027770833	.0013885417
6	.0013885417	.0027770833	.0016947917
7	.0009947917	.0019895833	.0018385417
8	.0005885417	.0011770833	.0016388021
9	.0002447917	.0004895833	.0007343750
10	.0000385417	.0000770833	.0001156250

TABLE IV. Coefficients for Deflections Produced by a Parabolic Moment Diagram.



Segment	For d=0.25	For d=0.50	For d=0.75
1	.00025000	.00016667	.00008333
2	.00175000	.00116667	.00058333
3	.00439583	.00316667	.00158333
4	.00566667	.00616667	.00308333
5	.00616667	.01016667	.00508333
6	.00616667	.01233333	.00758333
7	.00566667	.01133333	.01058333
8	.00466667	.00933333	.01310417
9	.00316667	.00633333	.00950000
10	.00116667	.00233333	.00350000

TABLE V. Coefficient for Deflections Produced by a Triangular Moment Diagram.



Segment	For α = 0.25	For α = 0.50	For α = 0.75
1	.003750	.002500	.001250
2	.011250	.007500	.003750
3	.017500	.012500	.006250
4	.016250	.017500	.008750
5	.013750	.022500	.011250
6	.011250	.022500	.013750
7	.008750	.017500	.016250
8	.006250	.012500	.017500
9	.003750	.007500	.011250
10	.001250	.002500	.003750

TABLE VI. Coefficients for Deflections Produced by a Rectangular Moment Diagram.

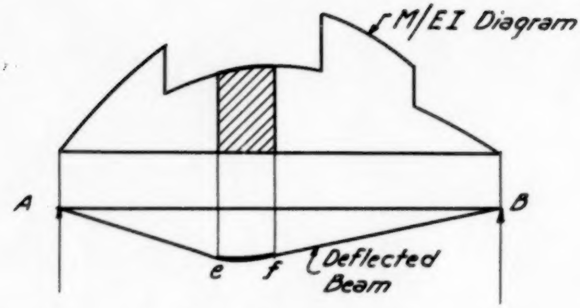


FIGURE 1. Action of One Flexural Segment.

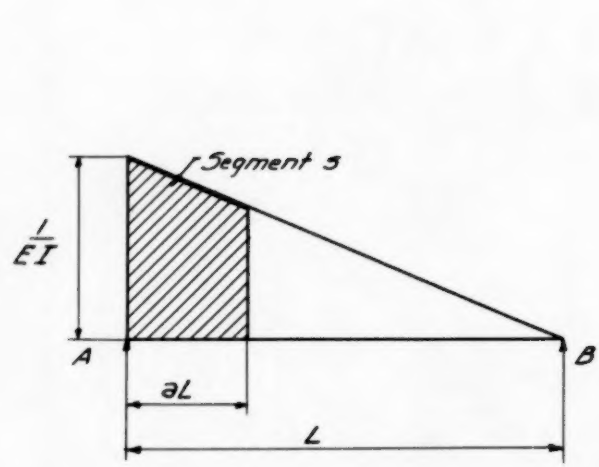


FIGURE 2. M/EI Diagram for Unit End Moment.

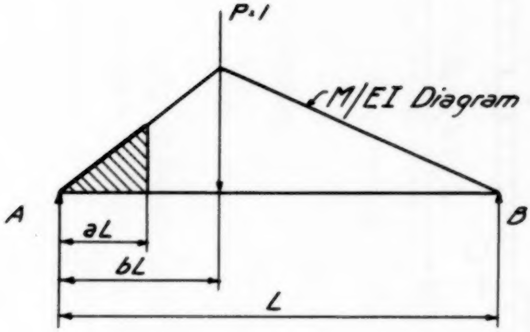


FIGURE 3. M/EI Diagram for Concentrated Load.

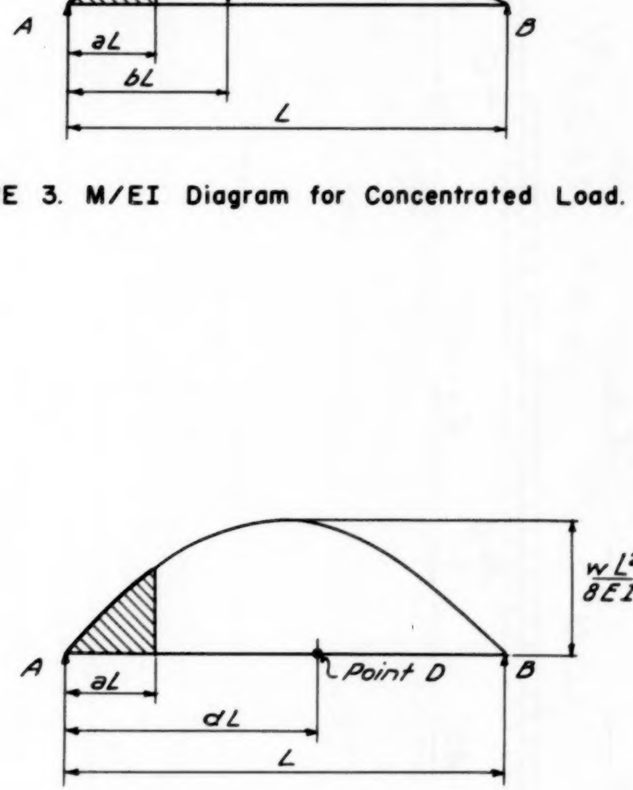


FIGURE 4. M/EI Diagram for Uniform Load.

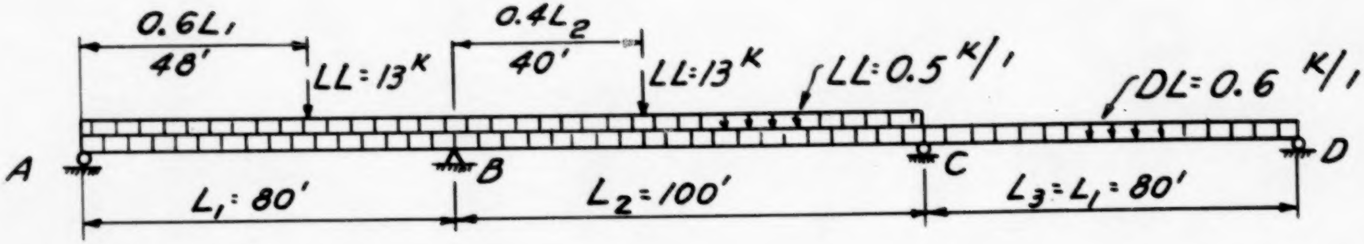


FIGURE 5. Three-span Continuous Beam and Loadings.

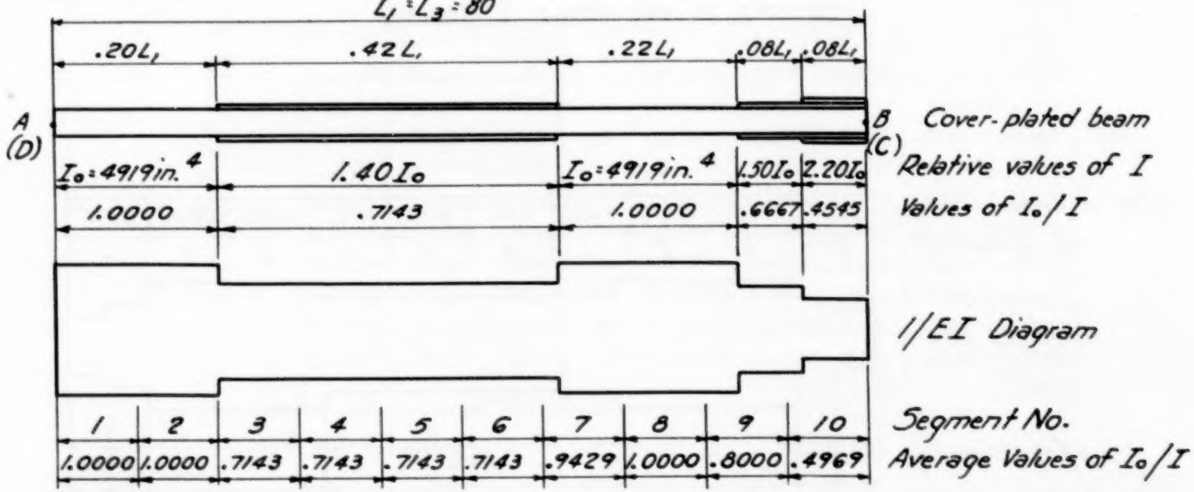


FIGURE 6. Properties of Span AB.

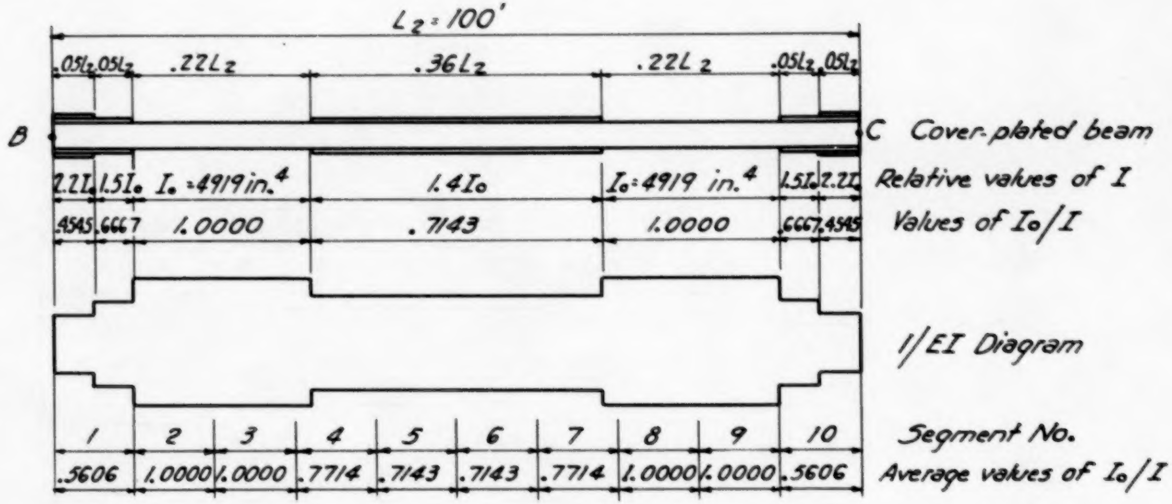


FIGURE 7. Properties of Span BC.

Span AB				Span BC					
Segment	I_o/I	For θ_B^B		Segment	I_o/I	For $\theta_B^B : \theta_C^C$		For $\theta_C^B : \theta_B^C$	
		Table I	θ_B^B			Table I	$\theta_B^B : \theta_C^C$	Table I	$\theta_C^B : \theta_B^C$
1	1.0000	.00033	.0003	1	0.5606	.09033	.0506	.00467	.0026
2	1.0000	.00233	.0023	2	1.0000	.07233	.0723	.01267	.0127
3	0.7143	.00633	.0045	3	1.0000	.05633	.0563	.01867	.0187
4	0.7143	.01233	.0088	4	0.7714	.04233	.0326	.02267	.0175
5	0.7143	.02033	.0145	5	0.7143	.03033	.0217	.02467	.0176
6	0.7143	.03033	.0217	6	0.7143	.02033	.0145	.02467	.0176
7	0.9429	.04233	.0399	7	0.7714	.01233	.0095	.02267	.0175
8	1.0000	.05633	.0563	8	1.0000	.00633	.0063	.01867	.0187
9	0.8000	.07233	.0579	9	1.0000	.00233	.0023	.01267	.0127
10	0.4969	.09033	.0449	10	0.5606	.00033	.0002	.00467	.0026
Σ			.2511	Σ			.2663		.1382

$$K_{BA} = \frac{1}{.2511} = 3.98\,\frac{EI_o}{L_1} \quad \text{(Equation 11)}$$

$$C_{BC} = C_{CB} = \frac{.1382}{.2663} = 0.519 \quad \text{(Equations 7 \& 8)}$$

$$K'_{BC} = K'_{CB} = \frac{.2663}{(.2663)^2 - (.1382)^2} = 5.14\,\frac{EI_o}{L_2}$$

(Equations 10 & 12)

FIGURE 8. Carry-over and Stiffness Factors.

Segment	I_o/I	For θ_B^P		For θ_B^W	
		Table II	θ_B^P	Table III	θ_B^W
1	1.0000	.00013	.00013	.00015	.00015
2	1.0000	.00093	.00093	.00098	.00098
3	0.7143	.00253	.00181	.00235	.00168
4	0.7143	.00493	.00352	.00398	.00284
5	0.7143	.00813	.00581	.00555	.00396
6	0.7143	.01213	.00867	.00678	.00484
7	0.9429	.01360	.01283	.00735	.00693
8	1.0000	.01120	.01120	.00698	.00698
9	0.8000	.00760	.00608	.00535	.00428
10	0.4969	.00280	.00139	.00218	.00108
Σ			.05237		.03372

M_B^F (Concentrated Load) $= 13.0(80)\left[.05237 \times 3.98\right] = 217^{\prime K}$ (Equation 23)

M_B^F (Uniform Load) $= 1.1(80)^2\left[.03372 \times 3.98\right] = \underline{944}^{\prime K}$ (Equation 24)

Total $M_B^F = 1161^{\prime K}$

FIGURE 9. Fixed End Moments Span AB.

Segment	I_o/I	For θ_B^P		For θ_C^P		For $\theta_B^W = \theta_C^W$	
		Table II	θ_B^P	Table II	θ_C^P	Table III	$\theta_B^W = \theta_C^W$
1	0.5606	.00280	.00157	.00020	.00011	.00218	.00122
2	1.0000	.00760	.00760	.00140	.00140	.00535	.00535
3	1.0000	.01120	.01120	.00380	.00380	.00698	.00698
4	0.7714	.01360	.01050	.00740	.00571	.00735	.00567
5	0.7143	.01213	.00867	.00987	.00705	.00678	.00484
6	0.7143	.00813	.00581	.00987	.00705	.00555	.00396
7	0.7714	.00493	.00380	.00907	.00700	.00398	.00307
8	1.0000	.00253	.00253	.00747	.00747	.00235	.00235
9	1.0000	.00093	.00093	.00507	.00507	.00098	.00098
10	0.5606	.00013	.00007	.00187	.00105	.00015	.00008
Σ			.05268		.04571		.03450

M_B^F (Concentrated Load) $= 13.0(100)[(.05268 \times 5.14) - (.04571 \times .519 \times 5.14)] = 194\ ^{1K}$ (Equation 19)

M_B^F (Uniform Load) $= 1.1(100)^2[(.03450 \times 5.14) - (.03450 \times .519 \times 5.14)] = 939\ ^{1K}$ (Equation 21)

Total $M_B^F = 1133\ ^{1K}$

M_C^F (Concentrated Load) $= 13.0(100)[(.04571 \times 5.14) - (.05268 \times .519 \times 5.14)] = 123\ ^{1K}$ (Equation 20)

M_C^F (Uniform Load) $= M_B^F$ $\quad 939\ ^{1K}$ (Equation 22)

Total M_C^F $1062\ ^{1K}$

FIGURE 10. Fixed End Moments Span BC.

Stiffnesses:

$K_{BA} = \frac{3.98}{80} = .0497$

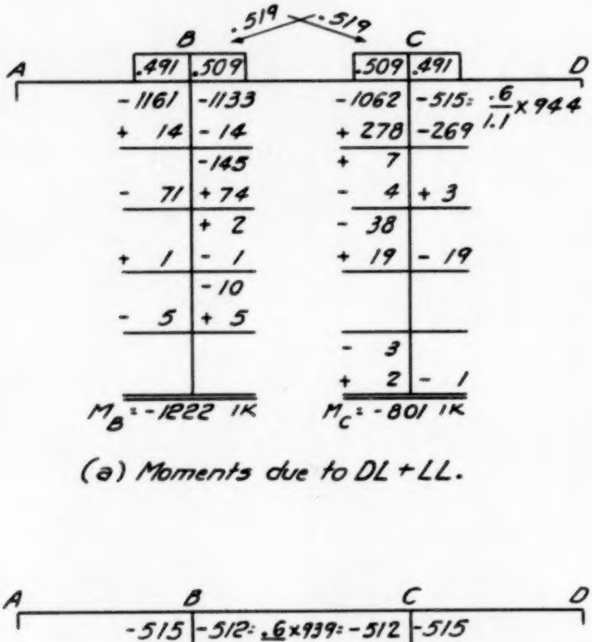
$K'_{BC} = \frac{5.14}{100} = .0514$

$.1011$

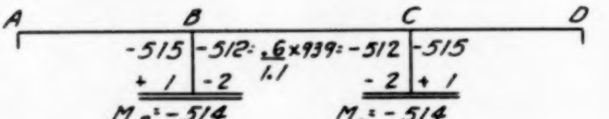
Distribution Factors:

$BA = CD = \frac{.0497}{.1011} = .491$

$BC = CB = \frac{.0514}{.1011} = .509$



(a) Moments due to DL + LL.



(b) Moments due to DL.

FIGURE 11. Moment Distribution Computations

Segment	Io/I	For Uniform Load		For Moment at B	
		Table IV	$\Delta_{0.5}$	Table V	$\Delta_{0.5}$
1	1.0000	.0000771	.000077	.0001667	.000167
2	1.0000	.0004896	.000490	.0011667	.001167
3	0.7143	.0011771	.000840	.0031667	.002260
4	0.7143	.0019896	.001416	.0061667	.004400
5	0.7143	.0027771	.001976	.0101667	.007250
6	0.7143	.0027771	.001976	.0123333	.008790
7	0.9429	.0019896	.001873	.0113333	.010690
8	1.0000	.0011771	.001177	.0093333	.009333
9	0.8000	.0004896	.000392	.0063333	.005060
10	0.4969	.0000771	.000038	.0023333	.001160
Σ			.010255		.050277

$$\Delta_{0.5}\downarrow = \frac{.010255 \times 0.6 \times 80^4}{30{,}000 \times 4919} = .001710 \times (12)^3 = 2.95'' \downarrow$$

$$\Delta_{0.5}\uparrow = \frac{.050277 \times 514 \times 80^2}{30{,}000 \times 4919} = .001121 \times (12)^3 = 1.94'' \uparrow$$

$$1.01'' \downarrow = \Delta_{0.5}$$

FIGURE 12. Dead Load Deflection at 0.5 Point of Span AB.

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